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EC 305 SPRING 2011

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1.

(a) Pr(X<=2)= Pr(X=0)+Pr(X=1)+Pr(X=2)=exp(-4)+4*exp(-4)+4^2*exp(-4)/2= 13exp(-4).
(b) Cov (X, Y) = 0, because X and Y are independent.
(c) Let X_i denote the i-th Saturday and X_i are independent identical Poisson Distribution with parameter 4. S_25= X_1 + X_2 + ... + X_25
E(S_25) = E(X_1) +E(X_2)+... + E(X_25)=25*4=100
Var(S_25)= Var(X_1)+ Var(X_2) +...+Var(X_25)=25*4=100
(d) The approximate distribution of S_25 is a normal distribution with mean 100 and variance 100 according to Central Limit Theorem.
(e) Pr(S_25>76.7)=Pr{ (S_25-25*4)/(5*2)> (76.7-100)/10}= Pr(Z>-2.33) =Pr(Z<=2.33)= 0.99.
(f) Similar to (d), T_25 is approximately a normal distribution with mean 25*(4+5)= 225 and variance 225. Therefore, Pr(T_25>225) =0.5.
(g) Pr(T_25>254.4)=Pr{ (T_25-225)/15> (254.4-225)/15}= Pr(Z>1.96) =1- Pr(Z<=1.96)= 0.025

- Note: For sufficiently large values of λ, (say λ>1000), the normal distribution with mean λ and variance λ (standard deviation$\sqrt{\lambda}$), is an excellent approximation to the Poisson distribution. If λ is greater than about 10, then the normal distribution is a good approximation if an appropriate continuity correction is performed, i.e., P($X \le x$), where (lower-case) x is a non-negative integer, is replaced by P($X \le x + 0.5$). $F_{\mathrm{Poisson}}(x;\lambda) \approx F_{\mathrm{normal}}(x;\mu=\lambda,\sigma^2=\lambda)$
  http://en.wikipedia.org/wiki/Poisson_distribution

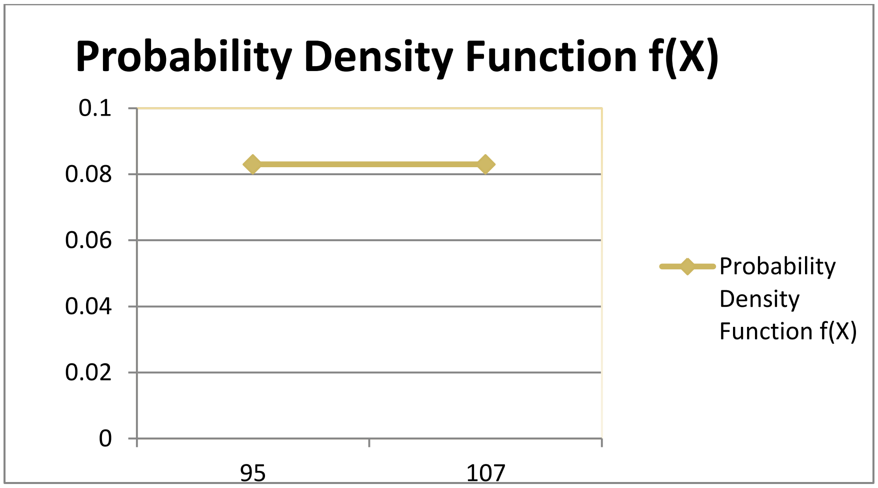
2.

(a) S_c = {(0H1H2H),(0H1H2M),(0H1H2L),(0H1M2H),(0H1M2M),(0H1M2L),(0H1L2H),(0H1L2M),(0H1L2L),(0M1H2H),(0M1H2M),(0M1H2L),(0M1M2H),(0M1M2M),(0M1M2L),(0M1L2H),(0M1L2M),(0M1L2L),(0L1H2H),(0L1H2M),(0L1H2L),(0L1M2H),(0L1M2M),(0L1M2L),(0L1L2H),(0L1L2M),(0L1L2L)}. There are 27 possible outcomes in total.
(b) X_c(0H1H2H)=3,X_c(0H1H2M)=2,X-c(0H1H2L)=2,X_c(0H1M2H)=2,X_c(0H1M2M)=1,X_c(0H1M2L)=1, X_c(0H1L2H)=2,X_c(0H1L2M)=1,X_c(0H1L2L)=1,X_c(0M1H2H)=2,X_c(0M1H2M)=1,X-c(0M1H2L)=1, X_c(0M1M2H)=1,X_c(0M1M2M)=0,X_c(0M1M2L)=0,X_c(0M1L2H)=1,X_c(0M1L2M)=0,X_c(0M1L2L)=0, X_c(0L1H2H)=2,X_c(0L1H2M)=1,X_c(0L1H2L)=1,X_c(0L1M2H)=1,X_c(0L1M2M)=0,X_c(0L1M2L)=0, X_c(0L1L2H)=1,X_c(0L1L2M)=0,X_c(0L1L2L)=0
(c) X_c's exact distribution is a Binomial distribution with n=3 and p=0.2.
(d) The approximate distribution for X_c100 is a normal distribution with mean 100*0.2=20 and variance 100*0.2*0.8=16.

- Note: A function can be defined by any mathematical condition relating each argument to the corresponding output value. If the domain is finite, a function *f* may be defined by simply tabulating all the arguments *x* and their corresponding function values *f*(*x*). More commonly, a function is defined by a formula, or (more generally) an algorithm — a recipe that tells how to compute the value of *f*(*x*) given any *x* in the domain.
- http://en.wikipedia.org/wiki/Central_limit_theorem
- http://en.wikipedia.org/wiki/Law_of_large_numbers

3.

(a) The pdf of X is a horizontal line from 95 to 107 with the vertical value of $f(X)= 1/12$.
The expectation of X is 101.(Refer to midterm 1 solution)



(b) The distribution of Y is a Uniform distribution on [290, 314]. $E(Y) =100 + 2 E(X) =302$

(c) $E(X\_bar)=101$

$Var(X\_bar)= 12/300=0.04= (0.2)^2$

$Pr(X\_bar<102) = Pr\{ (X\_bar-101)/0.2<5\}= Pr(z<5)=1$

(d) $Pr(X\_bar>a)=0.95=Pr(Z>-1.645)=Pr(Z>(a-101)/0.2)$

$a=101-0.2*1.645=100.671$

4.

(a) $T \sim Exp(0.5)$ $E(T)=2$ $Var(T)=4$

(b) $Pr(5<T<7)= Pr(T<7)- Pr(T<5) = [1- Pr(T>7)]- [1- Pr(T>5)]= Pr(T>5)- Pr(T>7)=exp(-2.5)- exp(-3.5)$

(c) $Pr(T>3+3 / T>3) =Pr(T>3) = exp(-1.5)$

(d) S100 is distributed N(nE(T), nV(T)), that is N(200, 400). Hence:

$Pr(S100>240)= Pr(Z>(240-200)/20)=Pr(Z>2)=.0228$

- Memorylessness: An important property of the exponential distribution is that it is memoryless. This means that if a random variable T is exponentially distributed. This says that the conditional probability that we need to wait, for example, more than another 10 seconds before the first arrival, given that the first arrival has not yet happened after 30 seconds, is equal to the initial probability that we need to wait more than 10 seconds for the first arrival. So, if we waited for 30 seconds and the first arrival didn't happen ($T > 30$), probability that we'll need to wait another 10 seconds for the first arrival ($T > 30 + 10$) is the same as the initial probability that we need to wait more than 10 seconds for the first arrival ($T > 10$). The fact that $Pr(T > 40 | T > 30) = Pr(T > 10)$ does not mean that the events $T > 40$ and $T > 30$ are independent. To summarize "memorylessness" of the probability distribution of the waiting time T until the first arrival means

$$(\text{Right})\ \Pr(T > 40 \mid T > 30) = \Pr(T > 10).$$